Optimal scaling laws for ductile fracture derived from strain-gradient microplasticity

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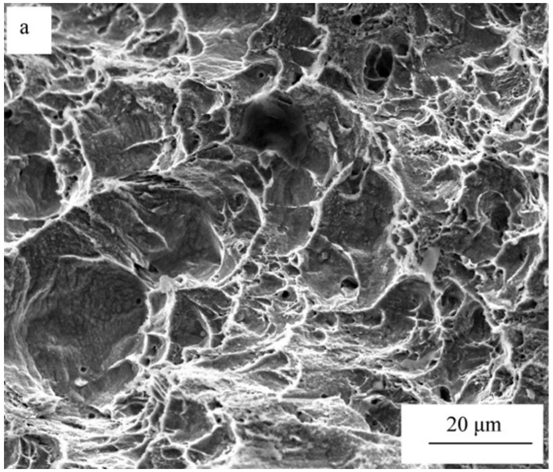
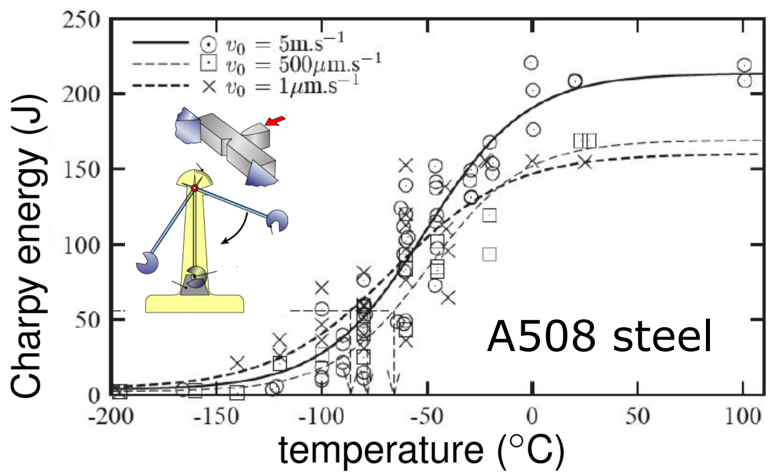
Ductile fracture of metals



SA333 steel, T=300K, $d\varepsilon/dt=3\times10^{-3}s^{-1}$
(S.V. Kamata, M. Srinivasa and P.R. Rao, *Mater. Sci. Engr*. A, **528** (2011) 4141.)

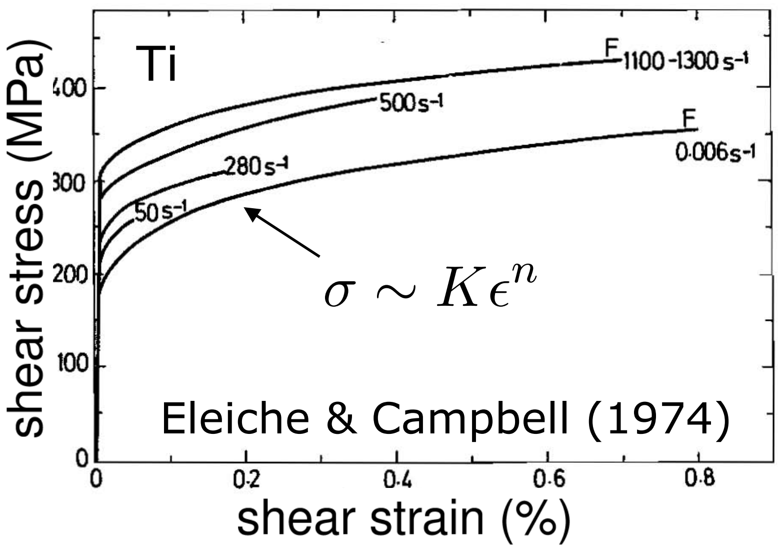


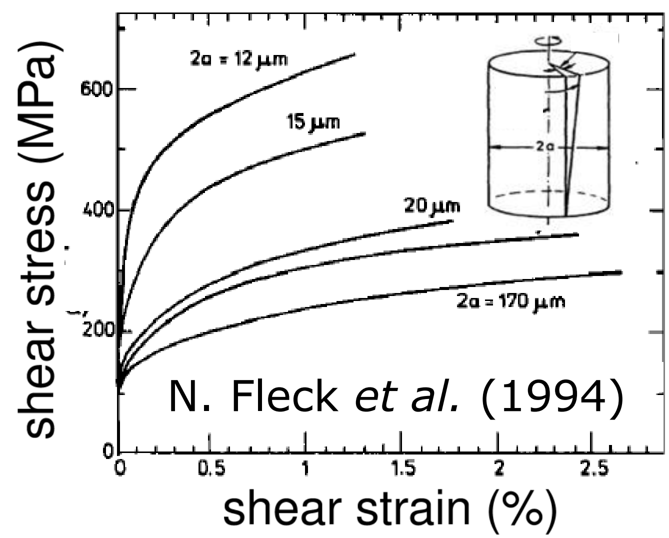
Tanguy, B., Besson, J., Piques, R. & Pineau, A., *Eng. Frac. Mechanics*, **72** (2005) 49.

- Ductile fracture in metals occurs by *void nucleation, growth and coalescence*
- Fractography of ductile fracture surfaces exhibits profuse *dimpling*, a vestige of microvoids
- Ductile fracture entails large amounts of *macrosocpic plastic deformation* and dissipation above TDB transition temperature
- *Ductile fracture is the quintessential multiscale phenomenon*!

Ductile fracture – Micromechanical basis

(plane-strain) specific fracture energy from *J*-testing (ASTM E813) → $J_{Ic} = \sigma_0 \ell$ ← dimensional analysis!





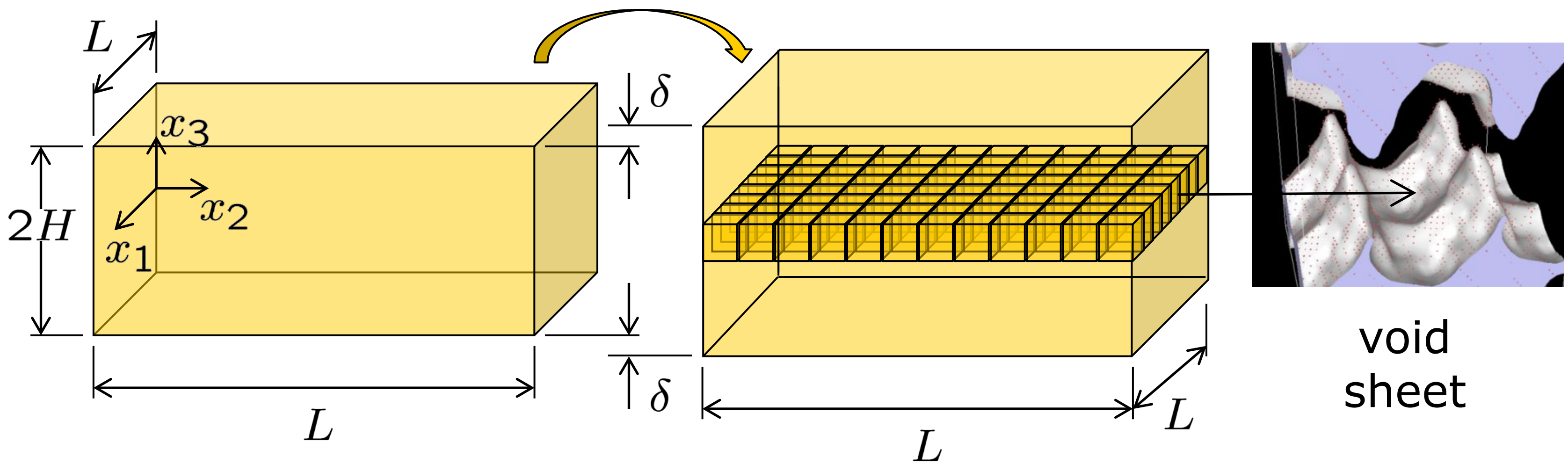
Plastic strength, material testing
ultimate strength $\sigma_0 = K/(n+1)$

Intrinsic characteristic length ℓ,
empirical material constant

N. A. Fleck, G.M. Muller, M.F. Ashby I and J.W. Hutchinson, "Strain-gradient plasticity: Theory and experiment", *Acta metall, mater*., **42**(2) (1994) 475.

From SGP to ductile fracture

- *Can ductile fracture be predicted by strain-gradient plasticity?*
- Model problem: Uniaxial extension of infinite slab



- Goals: *Bridge micro and macro scales analytically! Derive effective law that can be used in macroscale calculations!*
- Methodology and approach:
  - *Deformation theory of plasticity, finite kinematics, Lagrangian*
  - *Incompressible, rate-independent, rigid-plastic solid*
  - Micro-macro handshake: *Variational optimal scaling*

R. V. Kohn and S. Müller, Phil. Mag. A, **66**(5) (1992) 697-715.

Local deformation theory

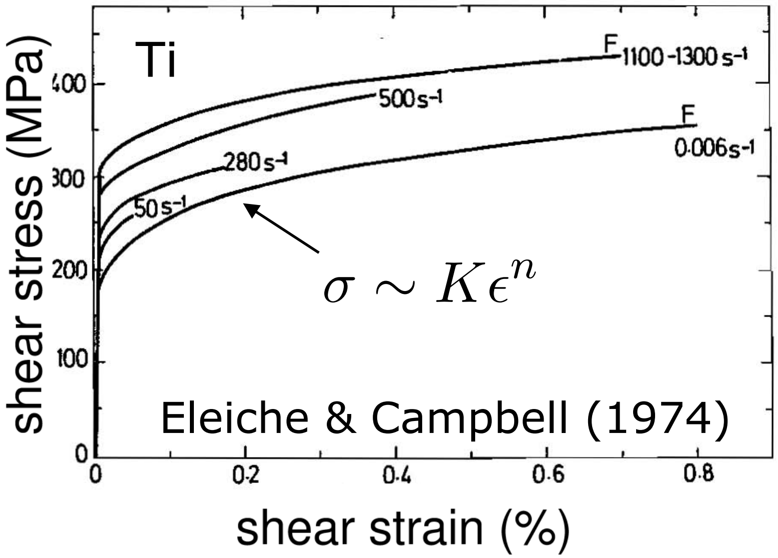
- Deformation theory: Minimize

$$E(y) = \int_\Omega W(Dy(x))\,dx,$$

$y : \Omega \to \mathbb{R}^d$, volume preserving.

- (Observed) growth of $W(F)$?
- Assume power-law hardening

$$\sigma \sim K\epsilon^n = K(\lambda - 1)^n.$$

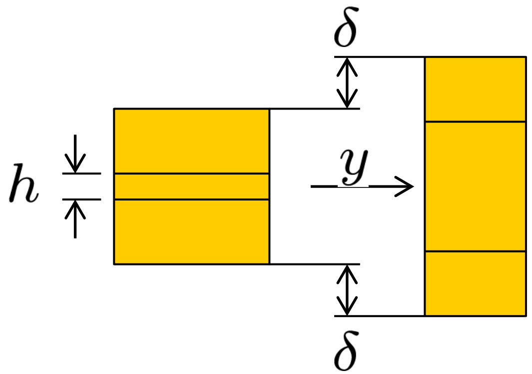


- Nominal stress: $\partial_\lambda W = \sigma/\lambda = K(\lambda - 1)^n/\lambda$.
- For large λ: $\partial_\lambda W \sim K\lambda^{n-1} \Rightarrow W \sim K\lambda^n$.
- Compare with $W(F) \sim |F|^p$, $p = n \in (0, 1)$.
- Considère analysis $\Rightarrow$ *Sublinear growth!* $(p < 1)$.

Necking of bars
Rittel et al. (2014)

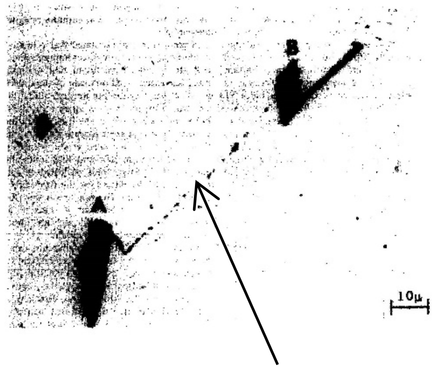
Armand Considère, *Annales des Ponts et Chaussées,* **9** (1885) 574-775.

Local deformation theory



localization of deformations

- Example: Uniaxial extension.
- Energy: $E_h \sim h\left(\frac{2\delta}{h}\right)^p$
- For $p < 1$: $\lim_{h\to 0} E_h = 0$



void sheet[1]

- Energies with sublinear growth relax to 0.
- For hardening exponents in the range of experimental observation, local plasticity yields no useful information regarding ductile fracture properties of materials.
- Need additional physics, structure…

[1]W.M. Garrison Jr. and N.R. Moody, "Ductile fracture", *J. Phys. Chem. Solids*, 48(II) (1987) 103.

Strain-gradient deformation theory

- The yield stress of metals is observed to increase in the presence of *strain gradients*.
- *Ansatz*: Minimize

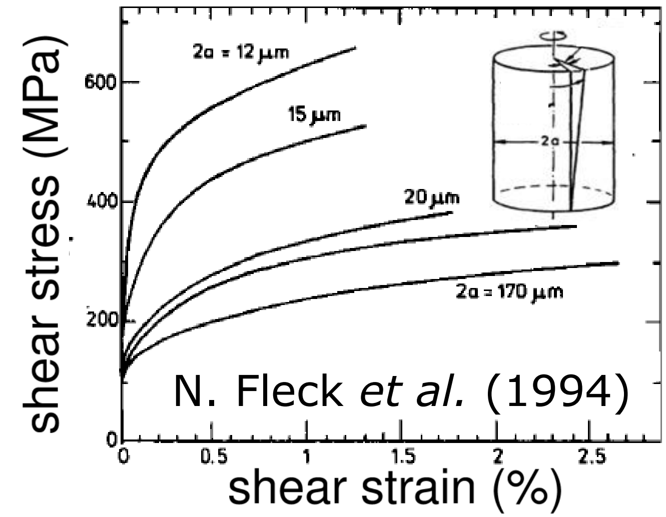
$$E(y) = \int_\Omega W\left(Dy(x), D^2y(x)\right)\, dx,$$

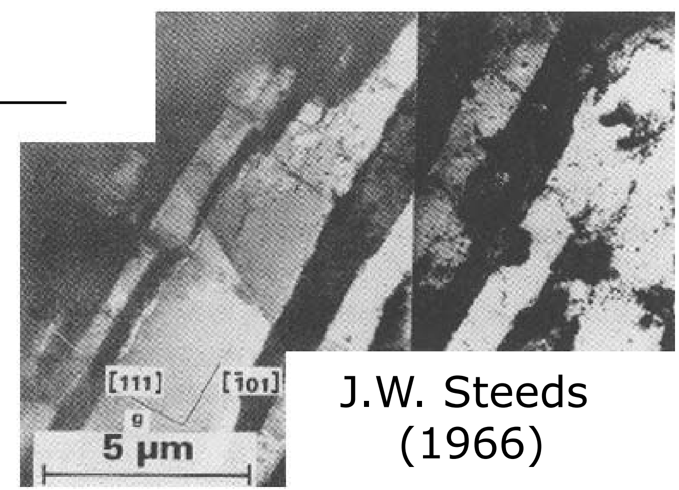
$y : \Omega \to \mathbb{R}^d$, volume preserving.

- Growth in $D^2y(x)$ *linear*, and

$$\ell = \frac{\mu b}{K} \quad \text{(\textit{characteristic length!})}$$

- *Can ductile fracture be understood as the result of a competition between local sublinear growth and strain-gradient plasticity?*





J.W. Steeds (1966)

Dislocation walls in copper (fence)

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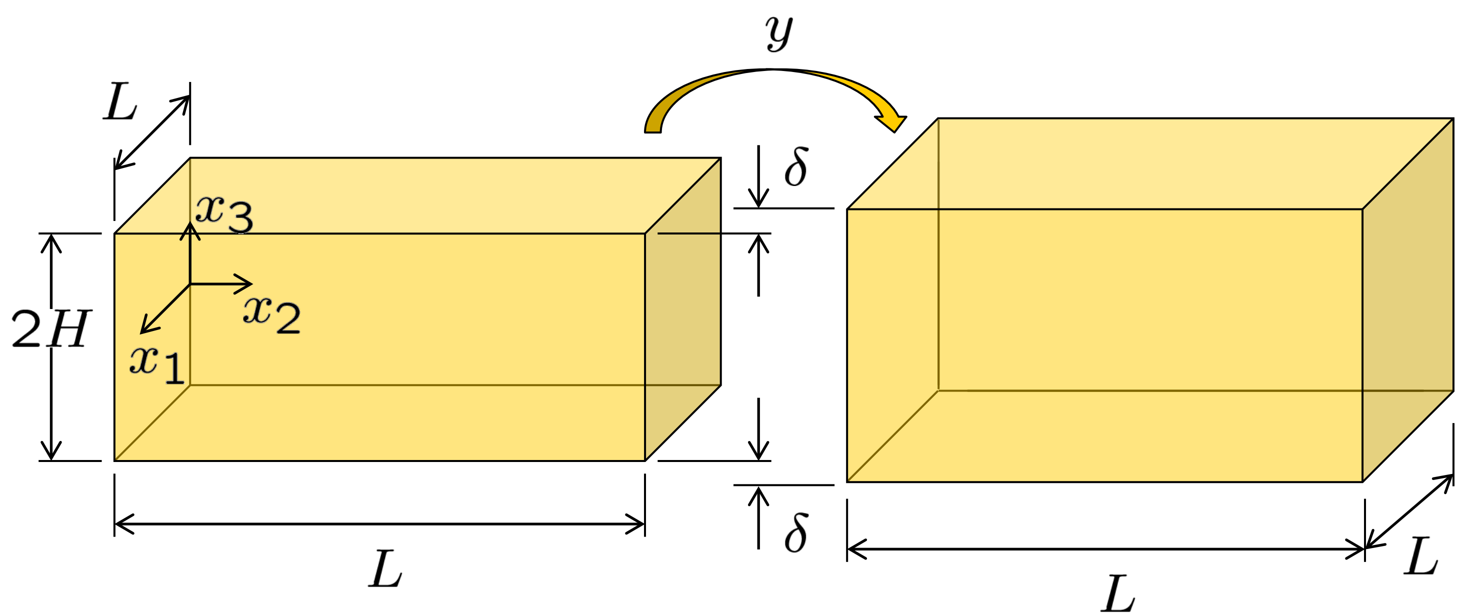
Ductile fracture: Optimal scaling



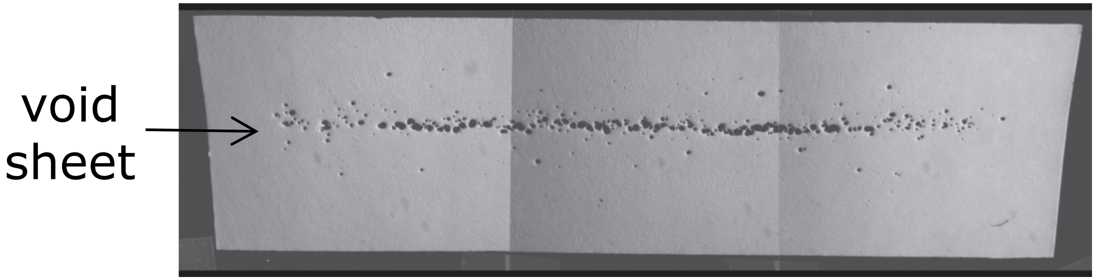
- Slab: $\Omega = [0, L]^2 \times [-H, H]$, in-plane periodic.
- Deformation $y \in W^{1,1}(\Omega; \mathbb{R}^3)$ and $Dy \in BV(\Omega; \mathbb{R}^{3\times 3})$,

$$\det Dy(x) = 1, \quad \text{a. e. in } \Omega.$$

- Uniaxial extension: $y_3(x_1, x_2, \pm H) = \pm(H + \delta)$.
- Growth: $E(y) \sim \int_\Omega \Big(|Dy(x)|^p + \ell |D^2 y(x)| \Big)\, dx$, $0 < p < 1$.

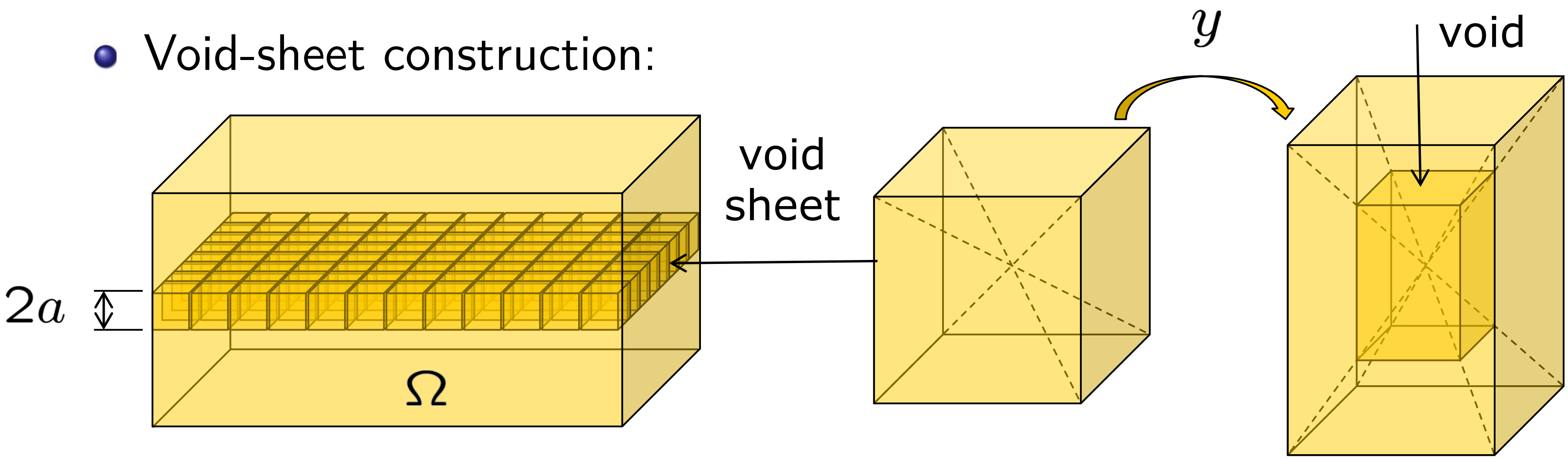
L. Fokoua, S. Conti & MO, *ARMA*, **212** (2014) 331–357.

Upper bound: Sketch of proof



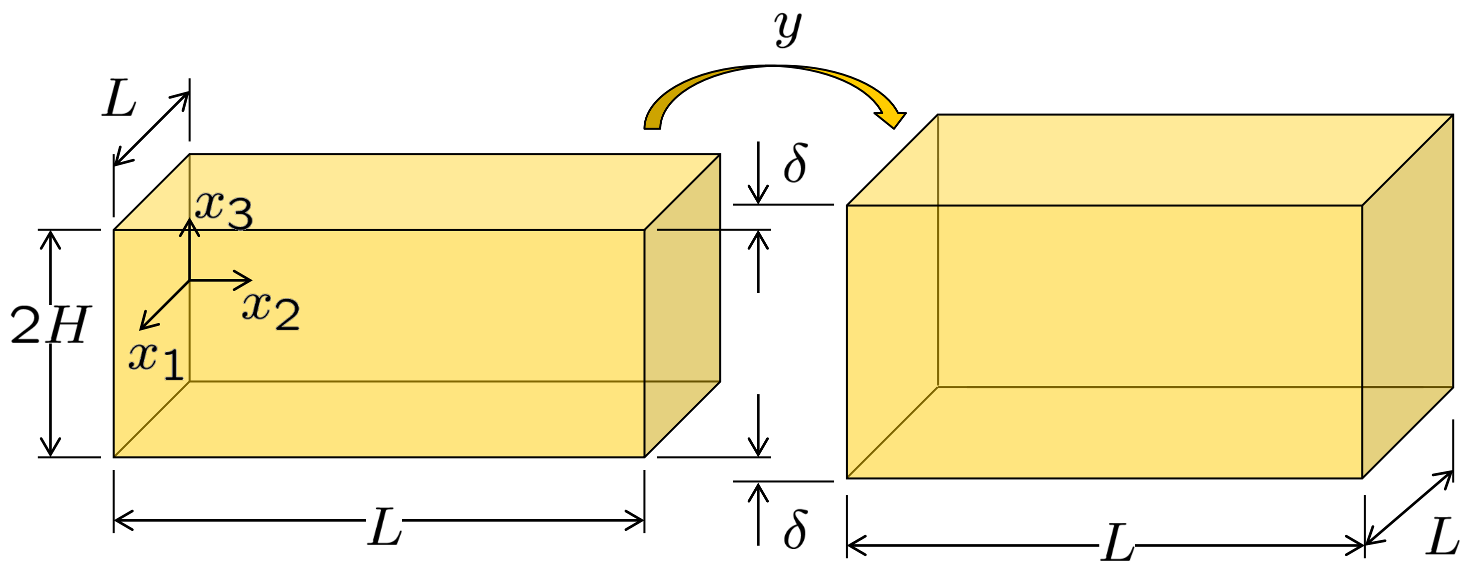
Heller, A., How Metals Fail, Science & Technology Review Magazine, LLNL, pp. 13-20, July/August, 2002

- Void-sheet construction:

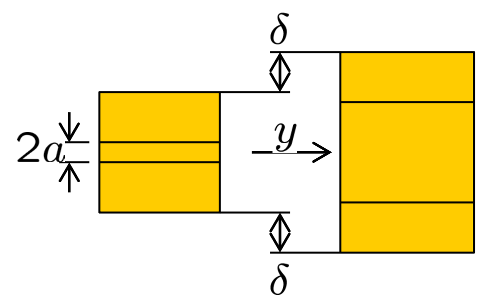


- Calculate, estimate: $E \leq CL^2 \left(a^{1-p}\delta^p + \ell\delta/a\right)$.
- Optimize thickness: $a_{\rm opt} \sim \ell^{\frac{1}{2-p}}\delta^{\frac{1-p}{2-p}}$ (coarsening).
- Optimal bound: $E \leq CL^2 \ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}}$.

Lower bound: Heuristics



- Ignore volume constraint, localize def to band of thickness $2a$.
- Trial energy: $E \sim \delta^p a^{1-p} + \ell(\delta/a)$.
- Optimize thickness: $a \sim \ell^{\frac{1}{2-p}} \delta^{\frac{1-p}{2-p}}$.
- Optimal energy: $E \sim \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$.



- To show: i) Same scaling can also be achieved by means of volume-preserving map; ii) scaling is optimal.

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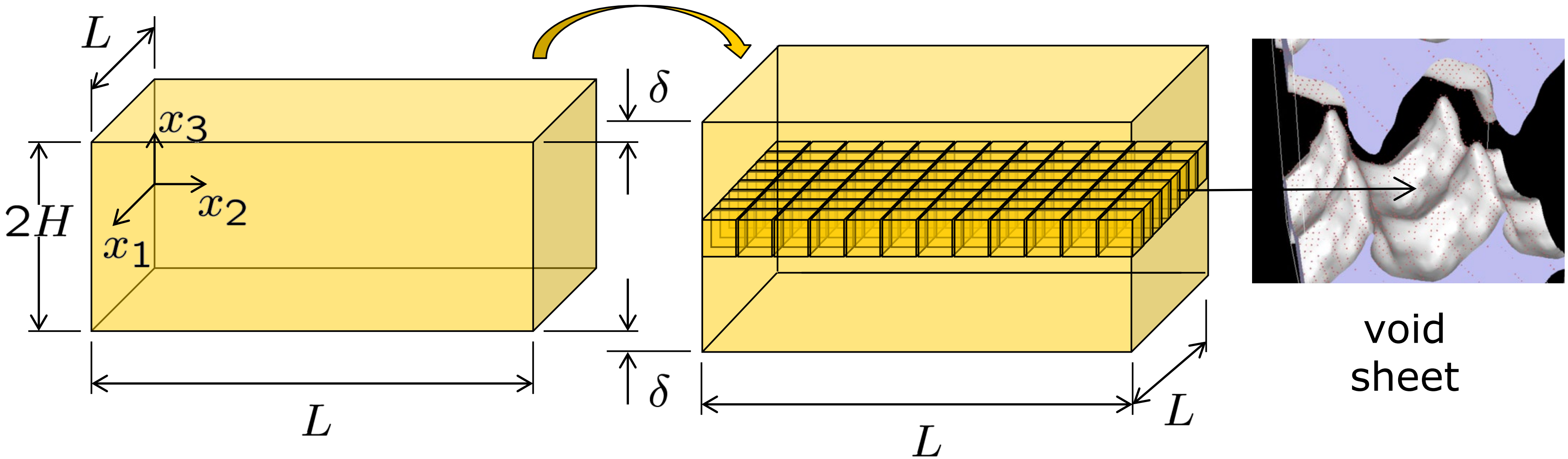
From SGP to ductile fracture



- Bounds on specific fracture energy:

$$C_L(p)\ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}} \leq J \leq C_U(p)\ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}}.$$

- Energy scales with power of opening displ (δ): Cohesive behavior!
- Bounds degenerate when the intrinsic length ℓ decreases to zero
- Theory provides a link between micro-plasticity (ℓ, constants) and macroscopic fracture (J).

From SGP to ductile fracture

Theorem (Upper bound)

Let $p \in [0,1)$, $\sigma \in (0,1)$, $H, L, \delta > 0$, *with* $0 \le \ell \le \delta \le L$. *Then, there is* $y \in W^{1,1}_{\text{loc}}(\mathbb{R}^3;\mathbb{R}^3)$ *such that* $y(x) = x \pm \delta e_3$ *for* $\pm x_3 \ge H$, y *is* $(0,L)^2$*-periodic in the first two variables and*

$$E(y) \le C L^2 \ell^{\frac{\sigma(1-p)}{1+\sigma-p}} \delta^{\frac{1-(1-\sigma)p}{1+\sigma-p}}.$$

The constant C *depends only on* p *and* σ.

Theorem (Lower bound)

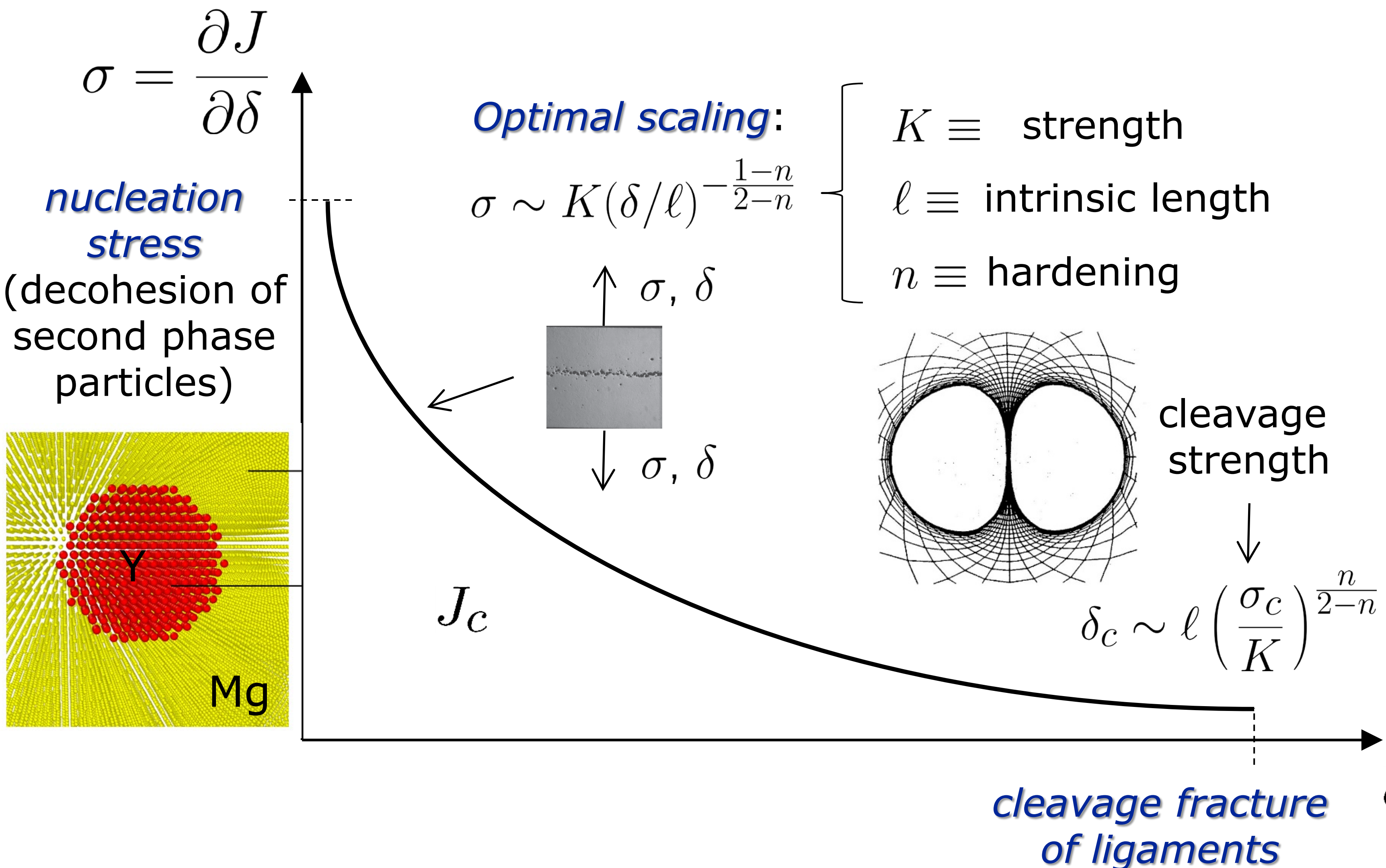
Let $p \in [0,1)$, $\sigma \in (0,1)$, $H, L, \delta > 0$, *let* $\Omega = (0,L)^2 \times (-H,H)$. *Then for sufficiently small* ℓ *we have*

$$C L^2 \ell^{\frac{\sigma(1-p)}{1+\sigma-p}} \delta^{\frac{1-(1-\sigma)p}{1+\sigma-p}} \le E(y),$$

for any $y : \Omega \to \mathbb{R}^3$ *such that* $y_3(x) = \pm(H+\delta)$ *for* $x_3 = \pm H$. *The constant* $C > 0$ *depends only on* p.

Ariza, M.P, Conti, S. & Ortiz, M., *Eur J Mech A Solids* (submitted).

Upscaling: Effective cohesive law



L. Fokoua, S. Conti & MO, *ARMA*, **212** (2014) 331–357.

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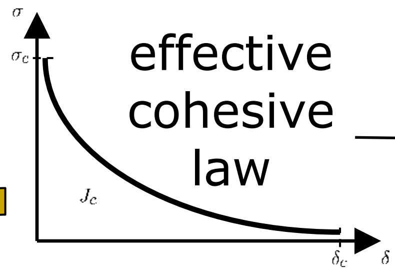
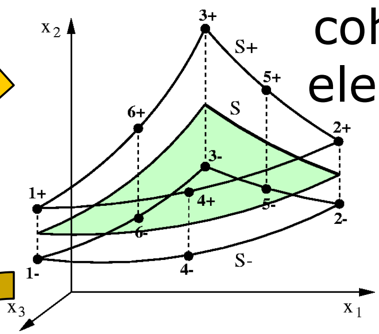
Spall fracture – Multiscale analysis

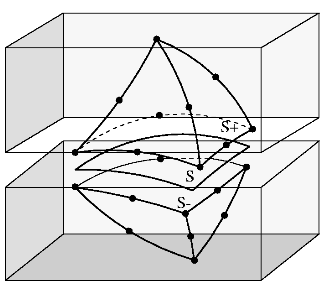
Fracture

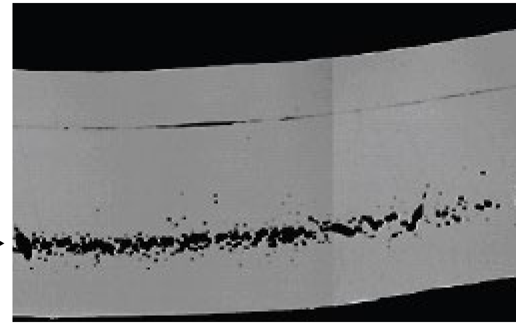


effective cohesive law



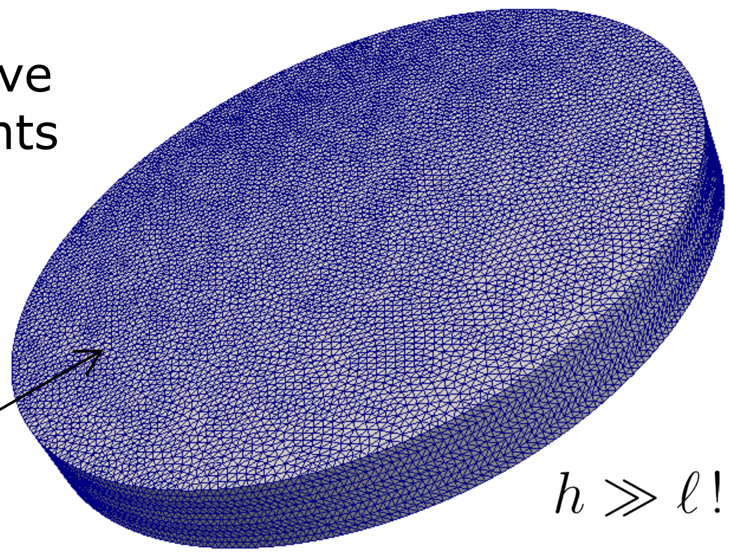
cohesive elements





Bulk

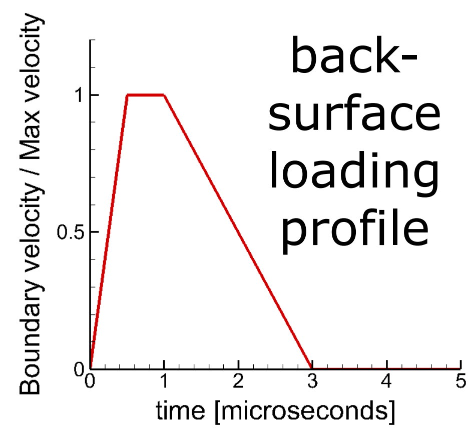
- J2 plasticity, power-law hardening
- *h= 0.49 mm*, 191,960 tets, 456,262 nodes



$h \gg \ell\,!$

Ni specimen
D = 50 mm, t = 4.95 mm

A. Pandolfi & MO (unpublished)



back-surface loading profile

Spall fracture – Multiscale analysis

- Ni specimen, D = 50 mm, t = 4.95 mm
- J2 plasticity, power-law hardening
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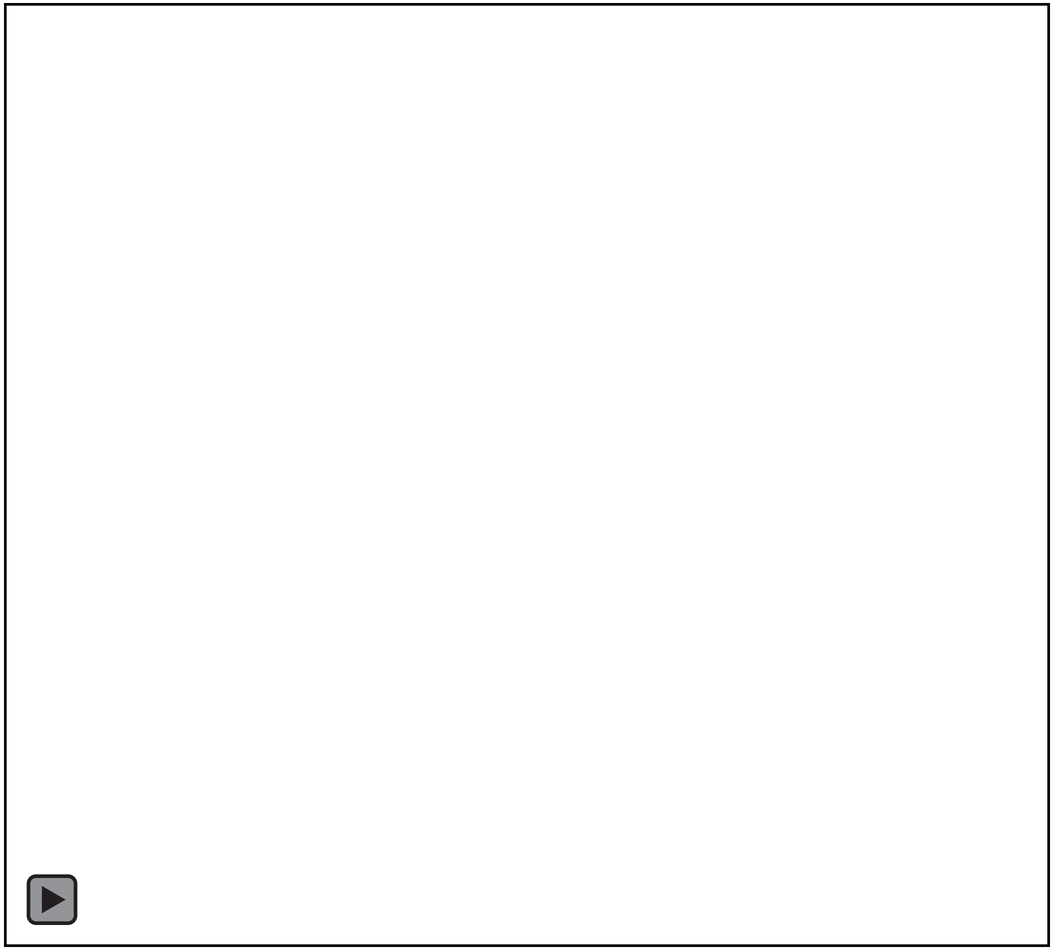
Spall fracture – Multiscale analysis



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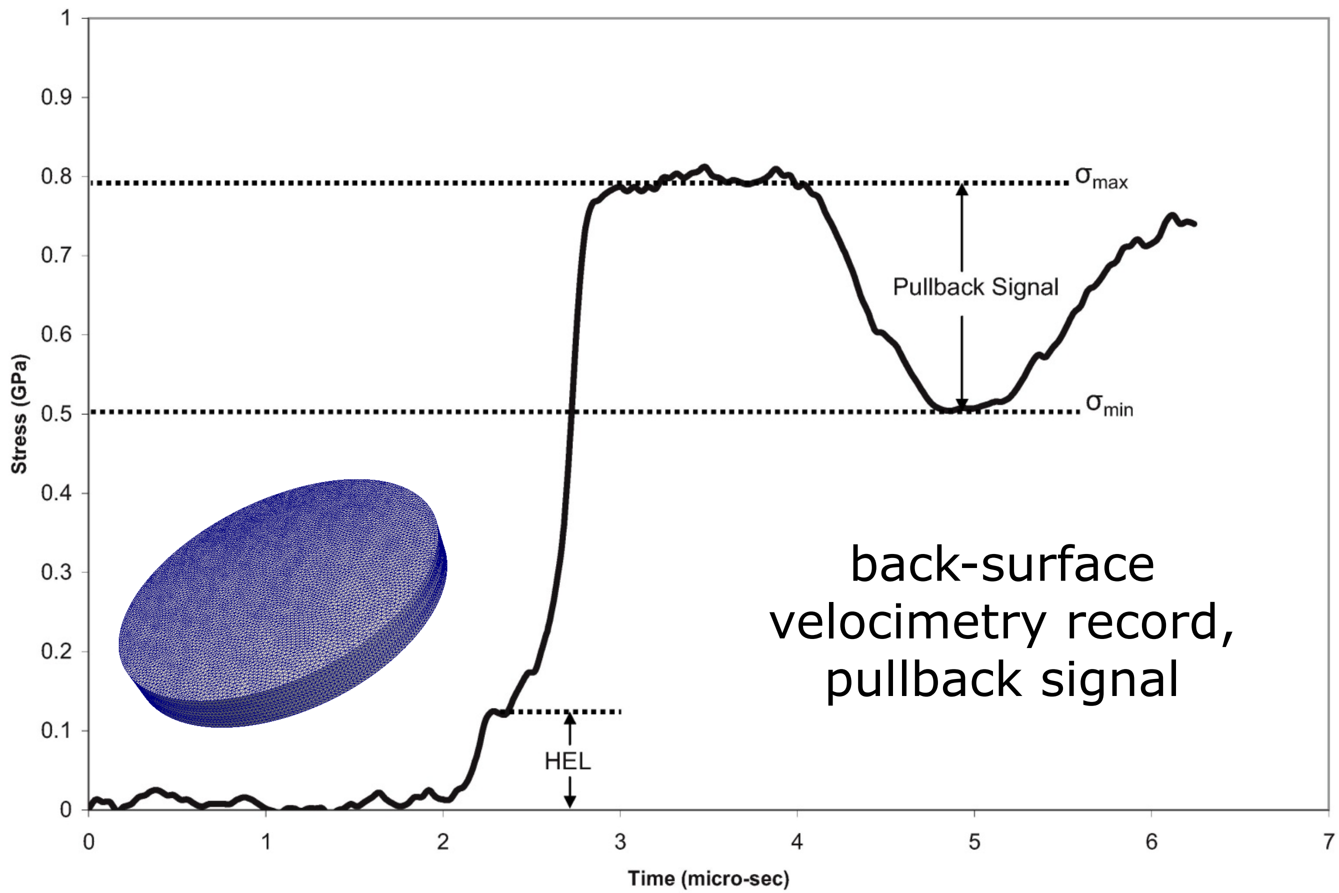
Spall fracture – Multiscale analysis



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A. Pandolfi & MO (unpublished)

What have we learned?

Pergamon

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Acta mater. Vol. 44, No. 10, pp. 3943–3954, 1996

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A SELF-CONSISTENT MODEL FOR CLEAVAGE IN THE PRESENCE OF PLASTIC FLOW

G. E. BELTZ[1], J. R. RICE[2], C. F. SHIH[3] and L. XIA[3]

[1]Department of Mechanical and Environmental Engineering, University of California, Santa Barbara, CA 93106-5070, [2]Division of Engineering and Applied Sciences and Department of Earth and Planetary Sciences, Harvard University, Cambridge, MA 02138 and [3]Division of Engineering, Brown University, Providence, RI 02912, U.S.A.

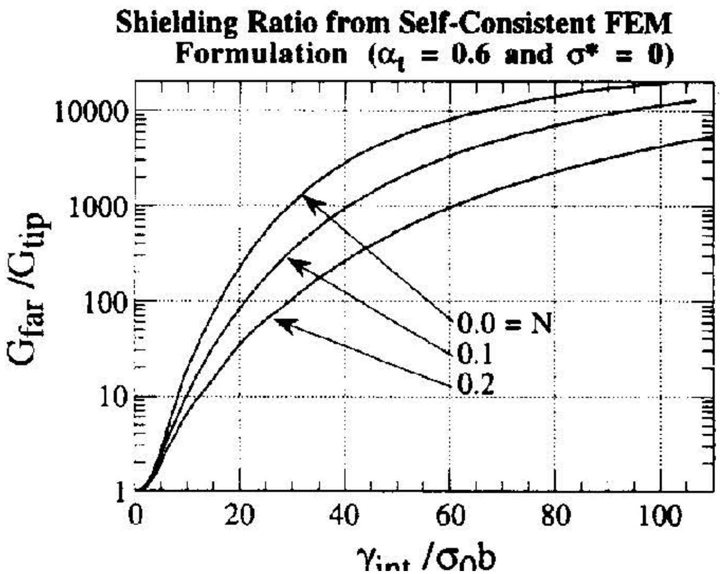
Abstract—A theory is proposed for cleavage in the presence of plastic flow, in circumstances which do not involve strong viscoplastic retardation of dislocation motion. We build upon recent notions that recognize the large disparity between relevant length scales involved in plastic flow processes around cracks in metals and on metal–ceramic interfaces.

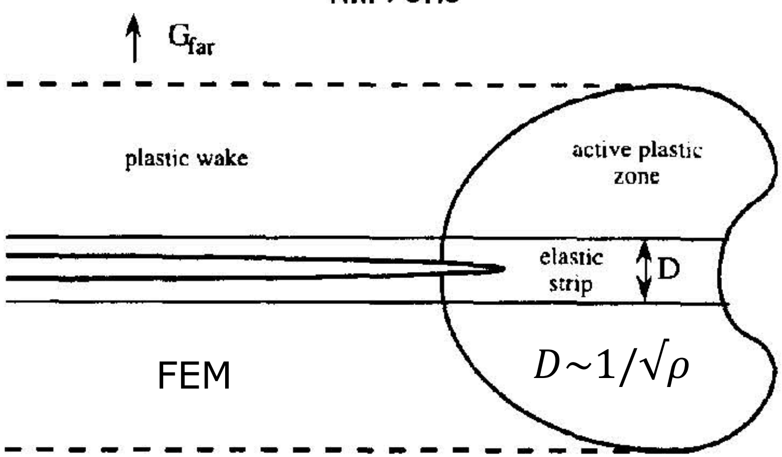
...

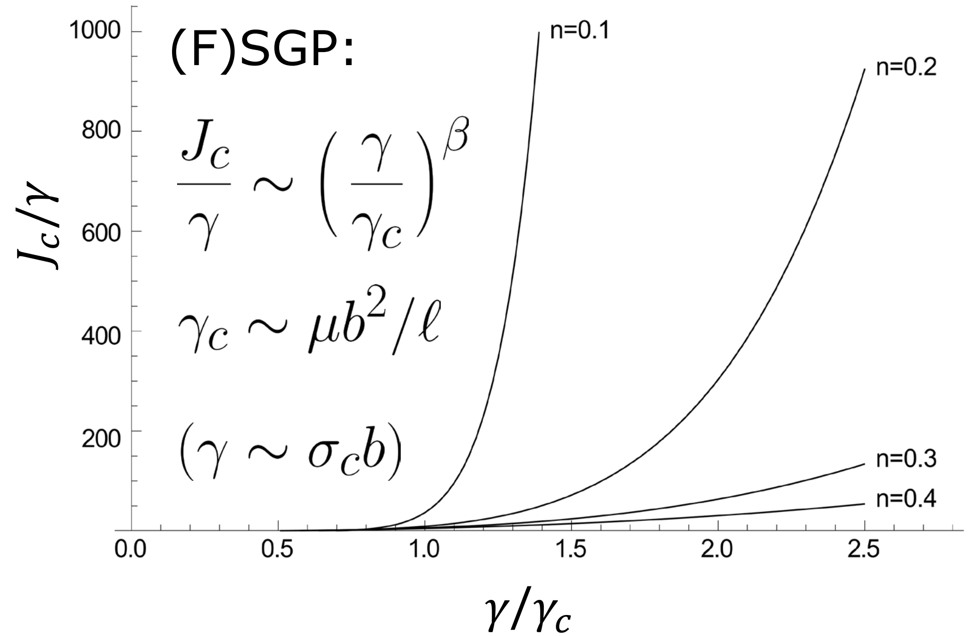
For steady-state crack growth to occur, it is found that the applied energy release rate G must generally be several orders of magnitude larger than the ideal work necessary to separate the interface, at least when D is taken as dislocation spacing. Furthermore, this "shielding" ratio is found to be strongly sensitive to the ideal work of fracture itself, as well as other material properties. *Copyright © 1996 Acta Metallurgica Inc.*

Michael Ortiz
CMCS 2023

What have we learned?







- J_c rises sharply above γ, provided $\gamma > \gamma_c$ (threshold)
- γ has ***gating effect*** on J_c
- (F)SGP + work hardening exponents < 1, explain ductile fracture, scaling

Beltz, G., Rice, J.R., Shih, C.F. & Xia, L., *Acta Mater*., **44**(10) (1996) 3943-3954.

Ariza, M.P, Conti, S. & Ortiz, M., *Eur J Mech A Solids* (submitted).

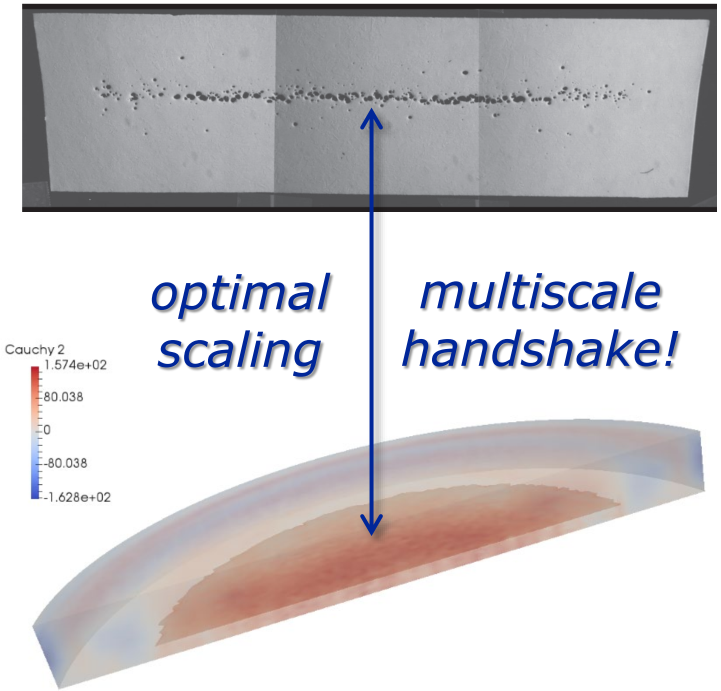
Ductile vs. brittle fracture

Material	K (MPa)	n	δ_c (mm)	b (nm)	μ (GPa)	γ (J/m^2)	ℓ (nm)	γ_c (J/m^2) s=0.2	s=0.3	s=0.4
OFHC Cu	400	0.29	6.47	0.255	44	1.725	8.14	0.305	0.180	0.127
Beryllium Cu	920	0.36	8.87	0.255	49	1.752	4.85	0.577	0.328	0.227
Al 1100	162	0.25	5.33	0.286	26	0.980	11.49	0.157	0.095	0.067
Al 356	358	0.10	1.77	0.286	27	0.980	2.18	0.806	0.519	0.382
Fe 310	1109	0.34	8.33	0.248	77	1.950	5.92	0.700	0.402	0.279
Fe 347	1343	0.31	7.35	0.248	74	1.950	4.34	0.915	0.534	0.374
Fe 303	1469	0.40	10.64	0.248	74	1.950	5.03	0.802	0.444	0.304
Fe 304	1319	0.39	10.15	0.248	74	1.950	5.44	0.740	0.413	0.283
Ni Annealed	2702	0.31	6.99	0.249	76	2.280	2.13	1.908	1.120	0.786
Ni Rene 41	2340	0.20	3.90	0.249	61	2.280	1.39	2.277	1.410	1.017
Ni K-Monel	1865	0.20	3.90	0.249	61	2.280	1.60	1.953	1.209	0.872

Table 1
Material constants for OFHC Cu, Beryllium Cu, Al 1100, Al 365, Fe 310, Fe 347, Fe 303, Fe 304, Ni, Ni Rene 41 and Ni K-Monel at room temperature. Sources: K, n, δ_c and μ from Warren and Reed (1963); b from Simon et al. (1992); $s = 0.2$ inferred for copper from Mu et al. (2014, 2016, 2017) by Dahlberg and Ortiz (2019); γ as tabulated in Hirth and Lothe (1968).

Ariza, M.P, Conti, S. & Ortiz, M., *Eur J Mech A Solids* (submitted).

Concluding remarks

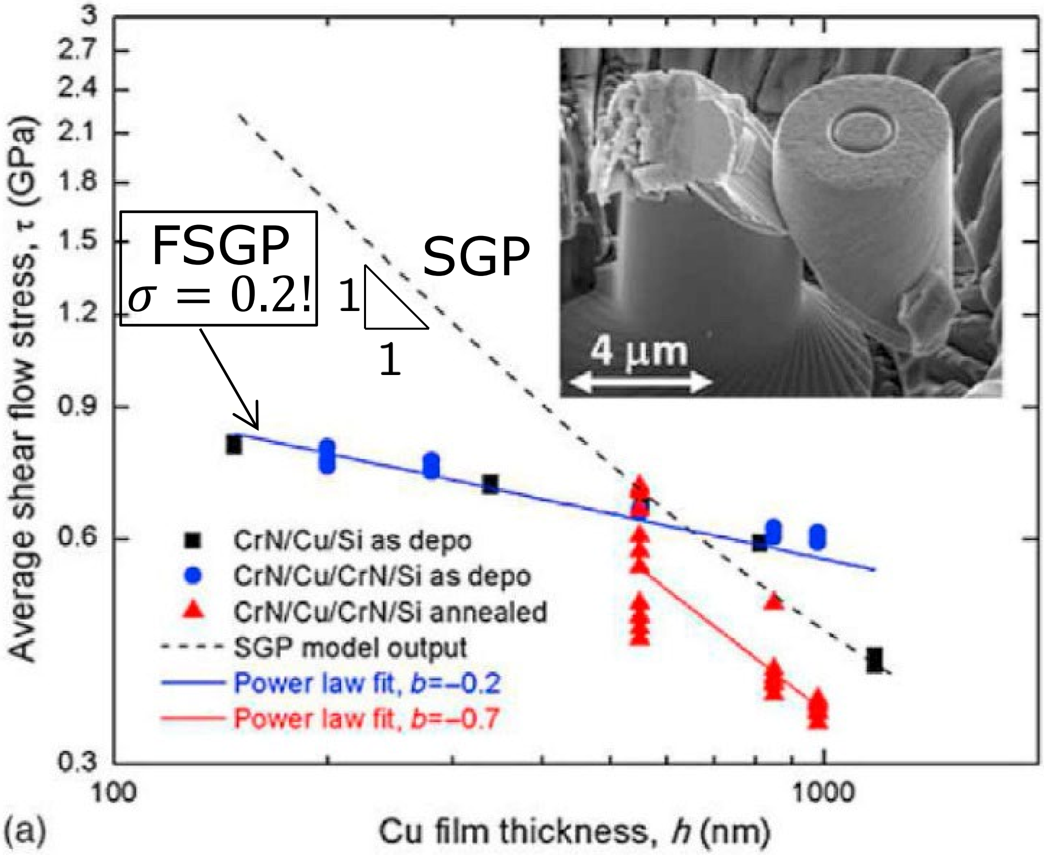


- *Strain-gradient plasticity* predicts *ductile fracture* as the result of a *competition* between *geometrical softening and strain-gradients.*
- *Optimal scaling* supplies an effective *analytical tool* for characterizing effective behavior at the macroscale (upscaling)
- Average normal stress *vs* opening displacement are found to *obey a power-law cohesive law*
- Exponents depend solely on the *growth properties* of the strain-gradient model, other details get 'buried' into constant factors
- *Effective cohesive law* represents *microscale mechanisms* (e.g., void sheets) in an effective sense at the *subgrid level*
- Can be embedded into standard *macroscale FE calculations*, e.g., as cohesive elements, in a *mesh-size insensitive* way

Concluding remarks

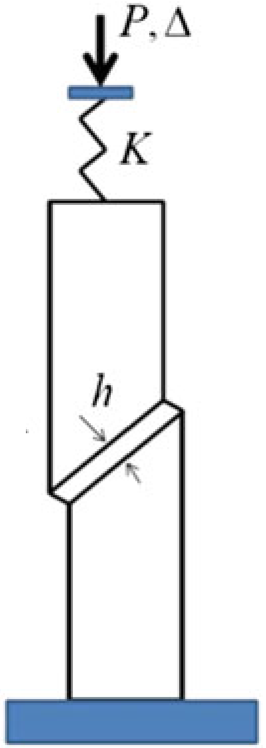
Thank you!

Fractional strain-gradient plasticity



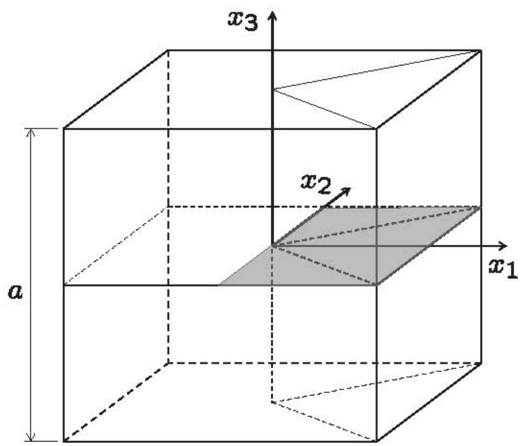
Shear flow stress as a function of thickness for Cu layers[1].
SGP model prediction shown as dashed line.
Insert shows SEM image of experimental setup.

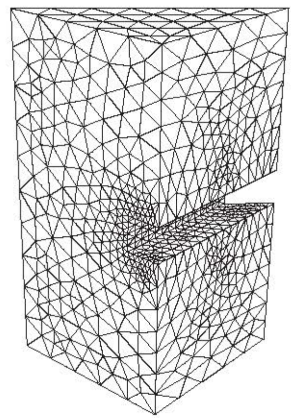
[1]Mu, Y., Zhang, X., Hutchinson, J.W., Meng, W.J., 2016. MRS Commun. Res. Lett. 20, 1–6.



Mu, Y., Zhang, X., Hutchinson, J.W., Meng, W.J., 2017. J. Mater. Res. 32 (8), 1421–1431.

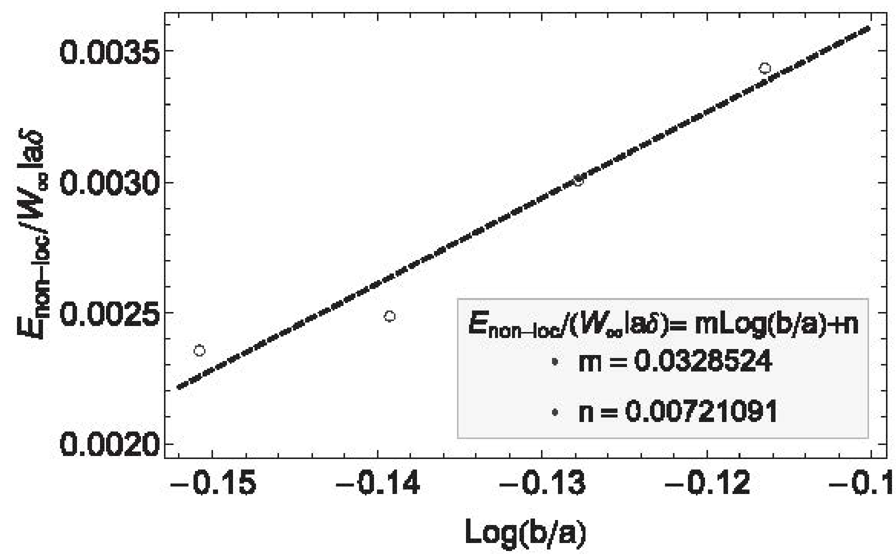
Optimal scaling – FE verification





- Nonlocal energy:

$$E_{\text{nonlocal}} = \sum_{\substack{\text{interior}\\ \text{element}\\ \text{faces}}} K\,\ell\,|\,[\![F]\!]\,|$$



S. Heyden, S. Conti and M. Ortiz, *Mechanics of Materials,* **90** (2015) 131-139.