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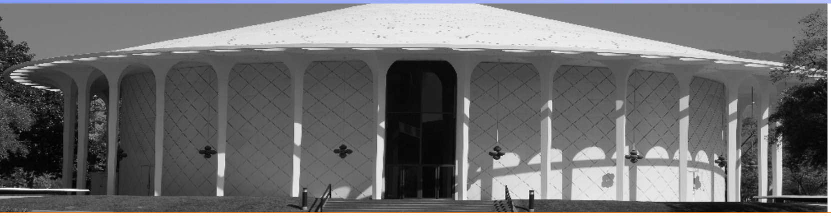

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Optimal scaling laws for ductile fracture derived from strain-gradient microplasticity

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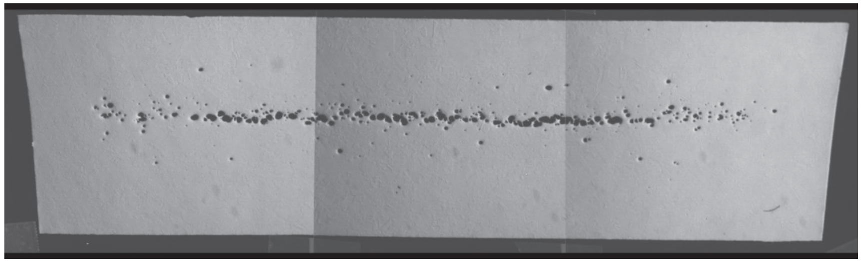
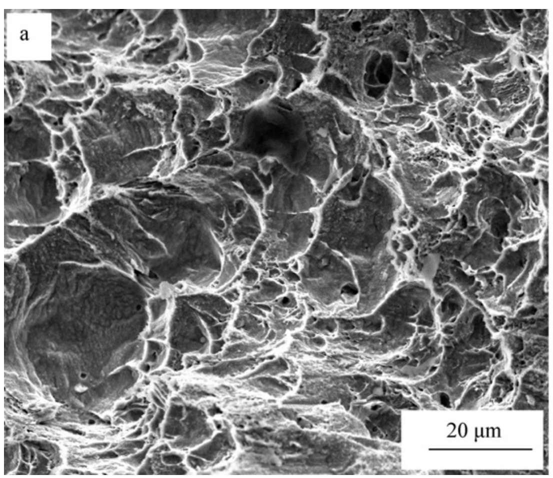
In collaboration with S. Conti (Universität Bonn)

CFRAC 20223
Prague, Czechia, 21-23 June 2023

Ductile fracture



Heller, A., How Metals Fail, Science & Technology Review Magazine, Lawrence Livermore National Laboratory, pp. 13-20, July/August, 2002



Fracture surface in SA333 steel, room temp., $d\varepsilon/dt=3\times10^{-3}s^{-1}$ (S.V. Kamata, M. Srinivasa and P.R. Rao, *Mater. Sci. Engr*. A, ***528*** *(2011) 4141–4146)*

- Ductile fracture in metals occurs by void nucleation, growth and coalescence
- Fractography of ductile-fracture surfaces exhibits profuse dimpling, vestige of microvoids
- Ductile fracture entails large amounts of plastic deformation and dissipation.
- Can ductile fracture be understood as the result of a competition between sublinear growth and strain-gradient plasticity?

Local deformation theory: Growth

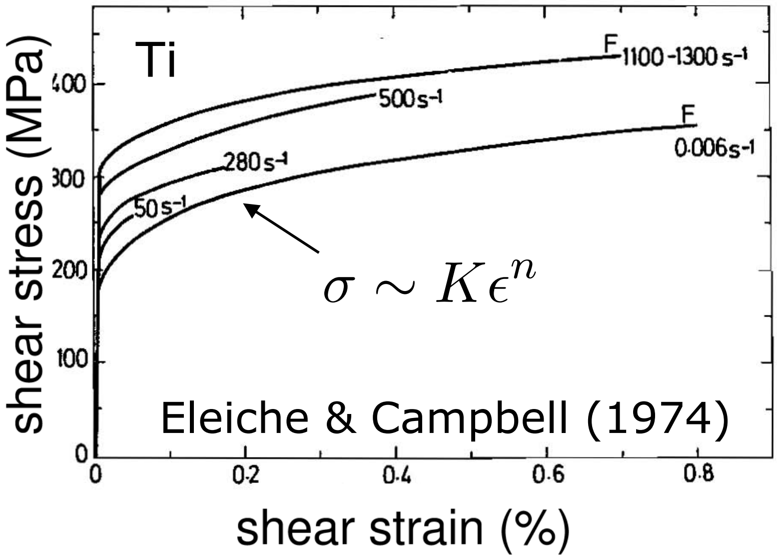
- Deformation theory: Minimize

$$E(y) = \int_\Omega W(Dy(x))\,dx,$$

$y : \Omega \to \mathbb{R}^d$, volume preserving.

- (Observed) growth of $W(F)$?
- Assume power-law hardening

$$\sigma \sim K\epsilon^n = K(\lambda - 1)^n.$$



- Nominal stress: $\partial_\lambda W = \sigma/\lambda = K(\lambda-1)^n/\lambda$.
- For large λ: $\partial_\lambda W \sim K\lambda^{n-1} \Rightarrow W \sim K\lambda^n$.
- Compare with $W(F) \sim |F|^p$, $p = n \in (0,1)$.
- Considère analysis $\Rightarrow$ *Sublinear growth!* $(p < 1)$.

Necking of bars
Rittel et al. (2014)

Armand Considère, *Annales des Ponts et Chaussées,* **9** (1885) 574-775.

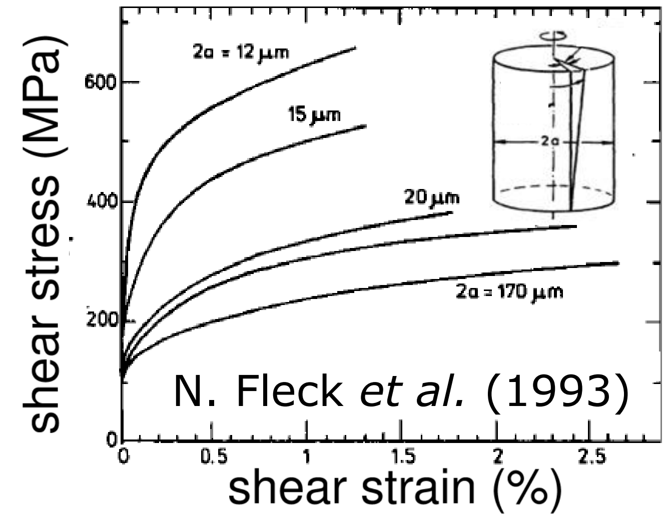
Strain-gradient plasticity

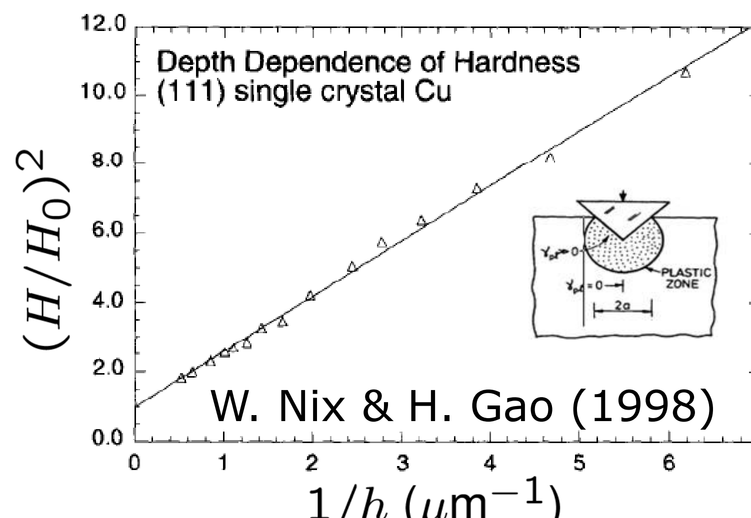
- Local energy relaxes to 0.
- Need additional physics!
- The yield stress of metals is observed to increase in the presence of *strain gradients*.
- *Ansatz*: Minimize

$$E(y) = \int_\Omega W\Big(Dy(x), D^2y(x)\Big)\, dx,$$

$y : \Omega \to \mathbb{R}^d$, volume preserving.

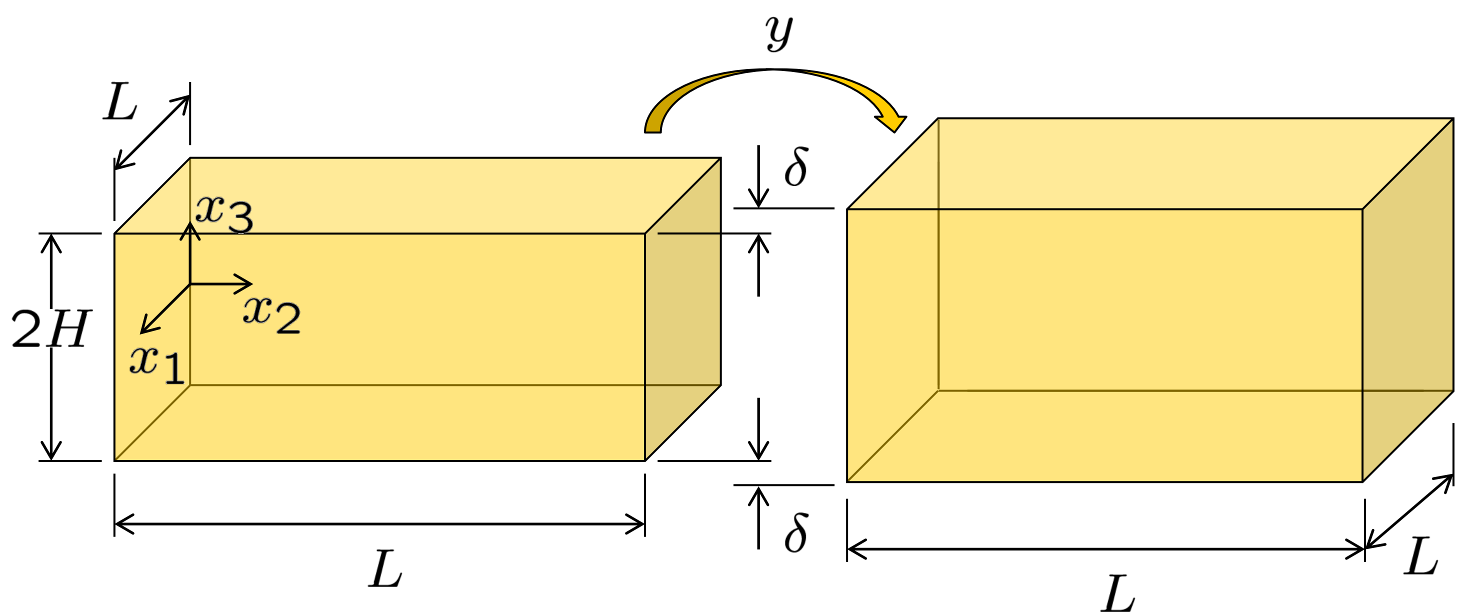
- Can ductile fracture be understood as the result of a competition between sublinear growth and strain-gradient plasticity?





Michael Ortiz
CFRAC 2023

Ductile fracture: Optimal scaling



- Slab: $\Omega = [0, L]^2 \times [-H, H]$, in-plane periodic.
- Deformation $y \in W^{1,1}(\Omega; \mathbb{R}^3)$ and $Dy \in BV(\Omega; \mathbb{R}^{3\times 3})$,

$$\det Dy(x) = 1, \quad \text{a. e. in } \Omega.$$

- Uniaxial extension: $y_3(x_1, x_2, \pm H) = \pm(H + \delta)$.
- Growth: $E(y) \sim \int_\Omega \Big(|Dy(x)|^p + \ell |D^2 y(x)| \Big)\, dx$, $0 < p < 1$.

Ductile fracture: Upper bound

Theorem (L. Fokoua, S. Conti & MO'2014)

Let $\Omega = L\,\mathbb{T}^2 \times (-H, H)$, $H > 1$, $\ell \in (0,1)$, $p \in (0,1)$, *and*

$$E(y) \sim \int_\Omega \Big(|Dy(x)|^p + \ell |D^2 y(x)| \Big)\, dx.$$

Fix $\delta > 0$. *For every* ℓ *sufficiently small, there is a map* $y : \Omega \to \mathbb{R}^3$ *such that* $y_3(x_1, x_2, \pm H) = \pm(H + \delta)$ *for all* $(x_1, x_2) \in L\,\mathbb{T}^2$ *and such that*

$$E(y) \leq C(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}},$$

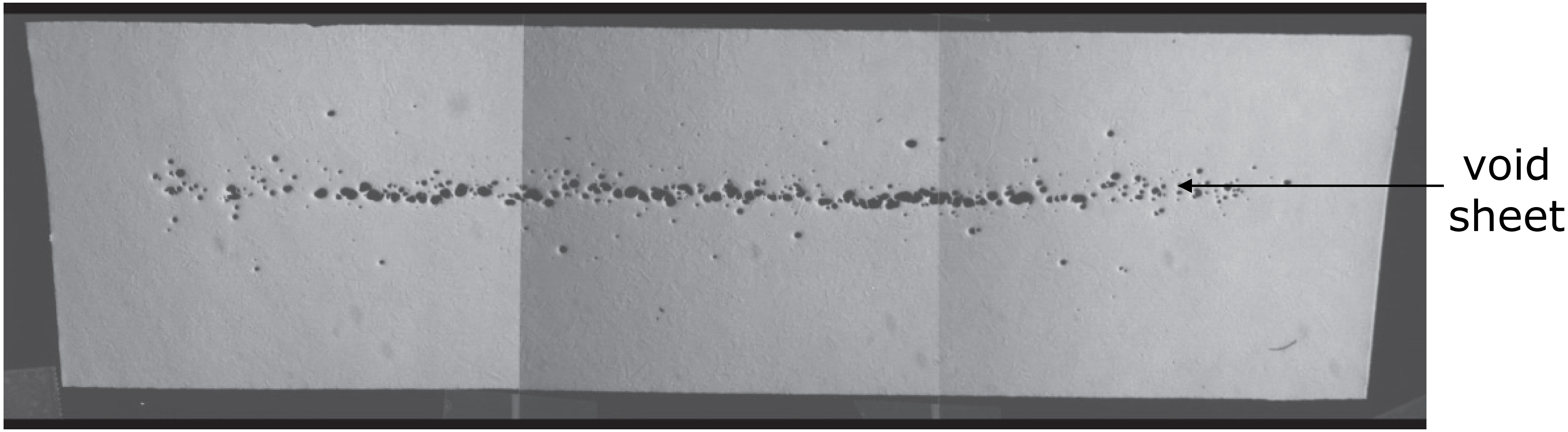
independently of H, *where, explicitly,*

$$C(p) = C \left((1-p)^{\frac{1}{2-p}} + (1-p)^{\frac{p-1}{2-p}} \right),$$

and $C > 0$ *is a universal constant.*

L. Fokoua, S. Conti & MO, *ARMA*, **212** (2014) 331–357.

Ductile fracture: Void sheets

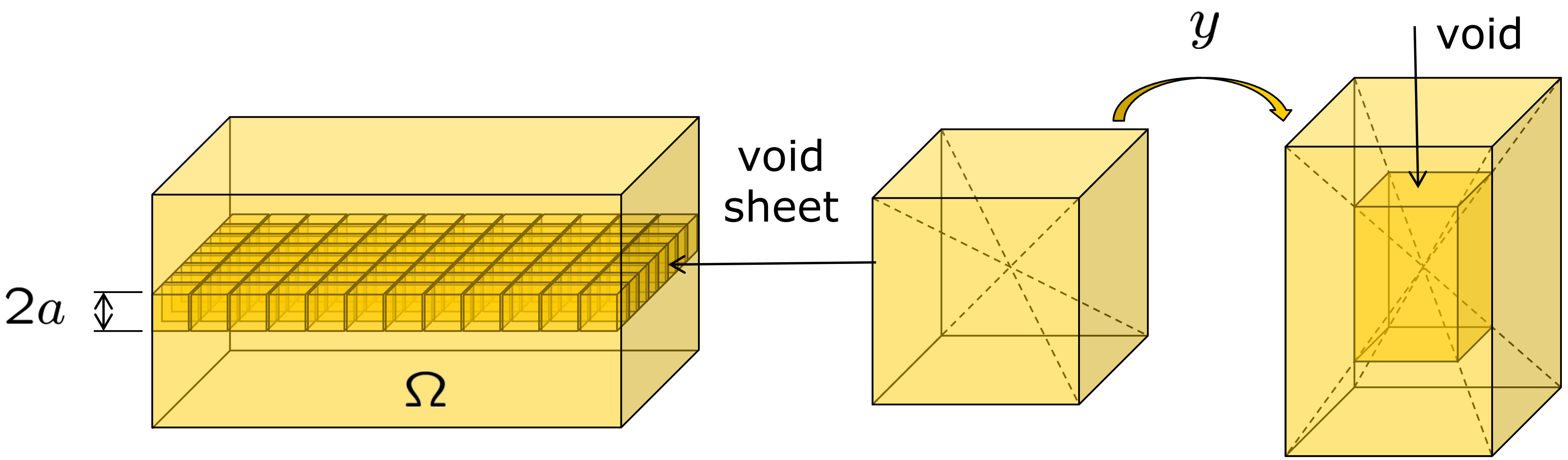


Heller, A., How Metals Fail, Science & Technology Review Magazine, Lawrence Livermore National Laboratory, pp. 13-20, July/August, 2002

- Volume conservation is restored by opening *voids* in the band, i. e., by means of a void-sheet construction.
- The void-sheet construction is related to constructions used in the mathematical literature of *cavitation* (Sverak'88; Ball'92; Müller & Spector'95; Conti & de Lellis'03; Henau & Mora-Corral'10).

Upper bound: Sketch of proof

- Void-sheet construction:



- Calculate, estimate: $E \leq CL^2 \left(a^{1-p}\delta^p + \ell\delta/a \right)$.
- Optimize thickness: $a_{\rm opt} \sim \ell^{\frac{1}{2-p}} \delta^{\frac{1-p}{2-p}}$ (coarsening).
- Optimal bound: $E \leq CL^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}}$. QED

Ductile fracture: Lower bound

Theorem (L. Fokoua, S. Conti & MO'2014)

Let $\Omega = L\,\mathbb{T}^2 \times (-H, H)$, $H > 1$, $\ell \in (0,1)$, $p \in (0,1)$, *and*

$$E(y) \sim \int_\Omega \Big(|Dy(x)|^p + \ell |D^2 y(x)| \Big)\, dx.$$

Fix $\delta > 0$. *For every* ℓ *sufficiently small, there is a map* $y : \Omega \to \mathbb{R}^3$ *such that* $y_3(x_1, x_2, \pm H) = \pm(H + \delta)$ *for all* $(x_1, x_2) \in L\,\mathbb{T}^2$ *and such that*

$$E(y) \geq C(p) L^2 \ell^{\frac{1-p}{2-p}} \delta^{\frac{1}{2-p}},$$

independently of H, *where, explicitly,*

$$C(p) = 2 \left(1 - \left(\frac{\sqrt{3}}{2} \right)^p \right) \left((1-p)^{\frac{1}{2-p}} + (1-p)^{\frac{p-1}{2-p}} \right).$$

L. Fokoua, S. Conti & MO, *ARMA*, **212** (2014) 331–357.

From micro-plasticity to ductile fracture

- Optimal (matching) upper and lower bounds:

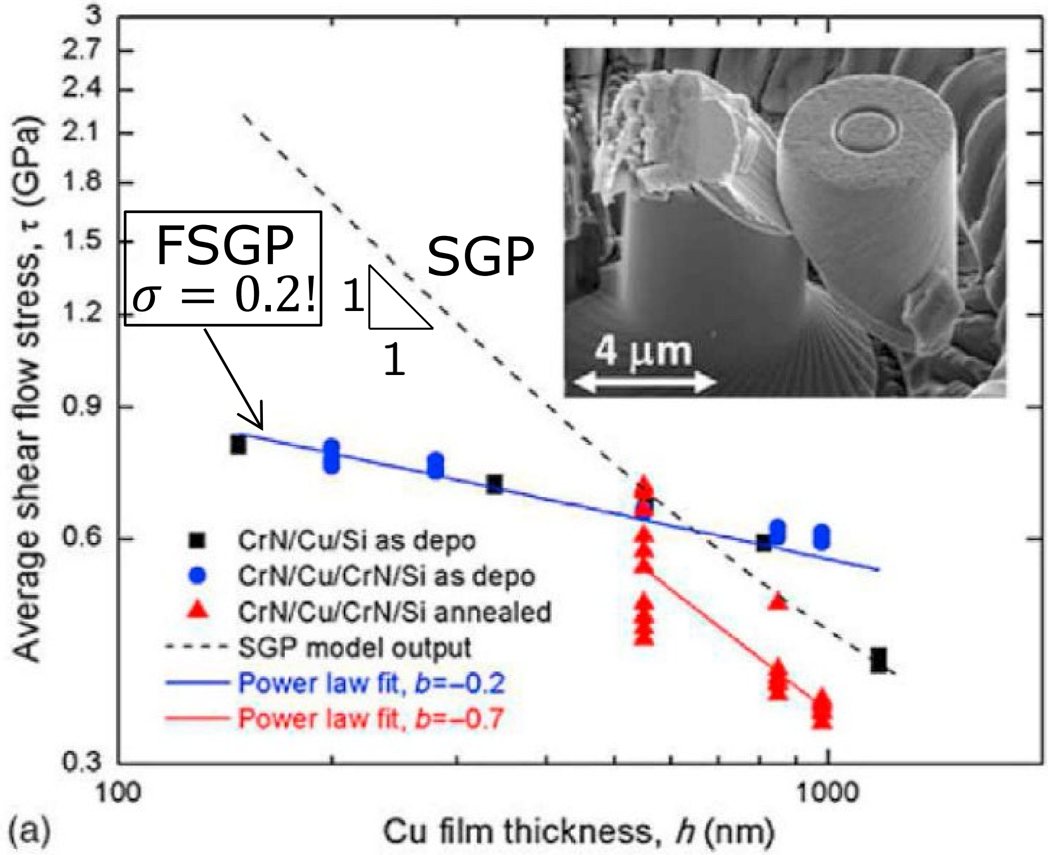
$$C_L(p)L^2\ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}} \leq \inf E \leq C_U(p)L^2\ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}}.$$

- Bounds apply to classes of materials having the same growth, specific model details immaterial
- Energy scales with area (L^2): Fracture scaling!
- Energy scales with power of opening displ (δ): Cohesive behavior!
- Bounds degenerate when the intrinsic length ℓ decreases to zero...
- Bounds on specific fracture energy:

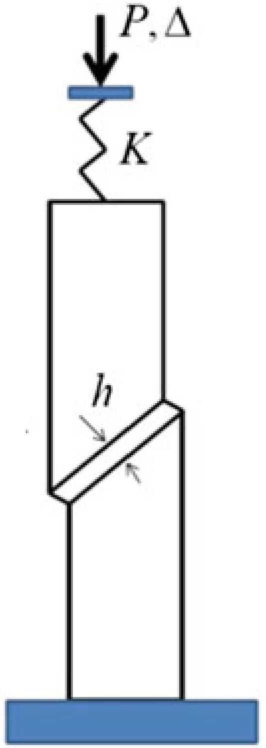
$$C_L(p)\ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}} \leq J_c \leq C_U(p)\ell^{\frac{1-p}{2-p}}\delta^{\frac{1}{2-p}}.$$

- Theory provides a link between micro-plasticity (ℓ, constants) and macroscopic fracture (J_c).

Fractional strain-gradient plasticity



Shear flow stress as a function of thickness for Cu layers[1].
SGP model prediction shown as dashed line.
Insert shows SEM image of experimental setup.



Mu, Y., Zhang, X., Hutchinson, J.W., Meng, W.J., 2017. J. Mater. Res. 32 (8), 1421–1431.

[1]Mu, Y., Zhang, X., Hutchinson, J.W., Meng, W.J., 2016. MRS Commun. Res. Lett. 20, 1–6.

Ductile fracture revisited: FSGP

Theorem (Upper bound, adapted from S. Conti & MO'2016)

Let $p \in [0,1)$, $\sigma \in (0,1)$, $H, L, \delta > 0$, *with* $0 \le \ell \le \delta \le L$. *Then, there is* $y \in W^{1,1}_{\text{loc}}(\mathbb{R}^3;\mathbb{R}^3)$ *such that* $y(x) = x \pm \delta e_3$ *for* $\pm x_3 \ge H$, y *is* $(0,L)^2$*-periodic in the first two variables and*

$$E(y) \le C L^2 \ell^{\frac{\sigma(1-p)}{1+\sigma-p}} \delta^{\frac{1-(1-\sigma)p}{1+\sigma-p}} .$$

The constant C *depends only on* p *and* σ.

Theorem (Lower bound, adapted from S. Conti & MO'2016)

Let $p \in [0,1)$, $\sigma \in (0,1)$, $H, L, \delta > 0$, *let* $\Omega = (0,L)^2 \times (-H,H)$. *Then for sufficiently small* ℓ *we have*

$$C L^2 \ell^{\frac{\sigma(1-p)}{1+\sigma-p}} \delta^{\frac{1-(1-\sigma)p}{1+\sigma-p}} \le E(y),$$

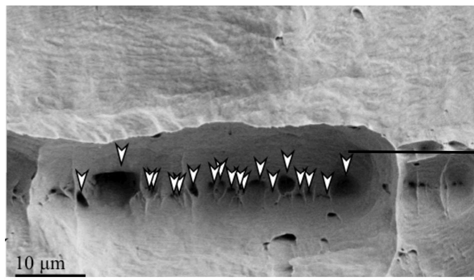
for any $y : \Omega \to \mathbb{R}^3$ *such that* $y_3(x) = \pm(H+\delta)$ *for* $x_3 = \pm H$. *The constant* $C > 0$ *depends only on* p.

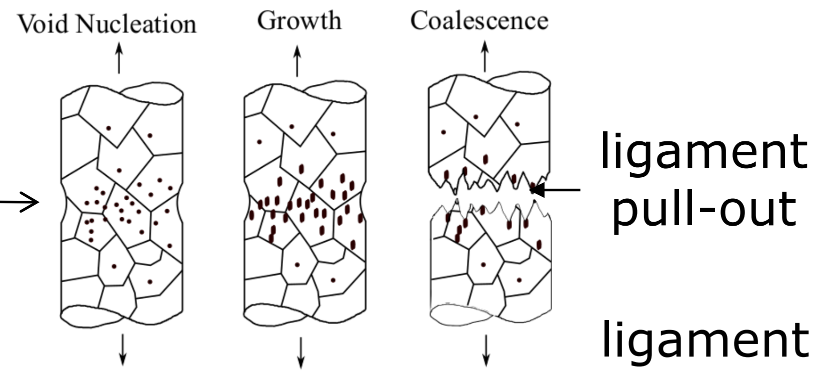
S. Conti & MO, *ARMA*, **219** (2016) 607–636

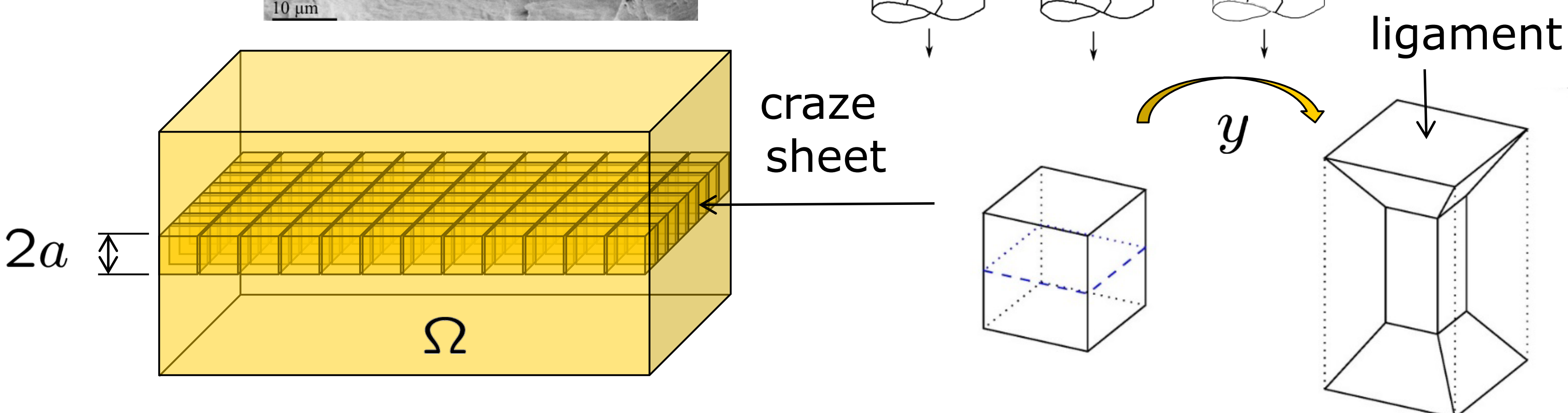
Upper bound: Sketch of proof

- Crazing construction:

P. Noell et al., SANDIA 2018-2431C







- Calculate, estimate: $E \le CL^2 \left(a^{1-p}\delta^p + \ell^\sigma \delta / a^\sigma \right).$
- Optimize thickness: $a_{\rm opt} \sim \ell^{\frac{\sigma}{\sigma+1-p}} \delta^{\frac{1-p}{\sigma+1-p}}$ (coarsening).
- Optimal bound: $E \le CL^2 \ell^{\frac{\sigma(1-p)}{1+\sigma-p}} \delta^{\frac{1-(1-\sigma)p}{1+\sigma-p}}.$ QED

S. Conti & MO, *ARMA*, **219** (2016) 607–636

What have we learned?

Pergamon

PII S-1359-6454(96)00047-X

Acta mater. Vol. 44, No. 10, pp. 3943–3954, 1996

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A SELF-CONSISTENT MODEL FOR CLEAVAGE IN THE PRESENCE OF PLASTIC FLOW

G. E. BELTZ[1], J. R. RICE[2], C. F. SHIH[3] and L. XIA[3]

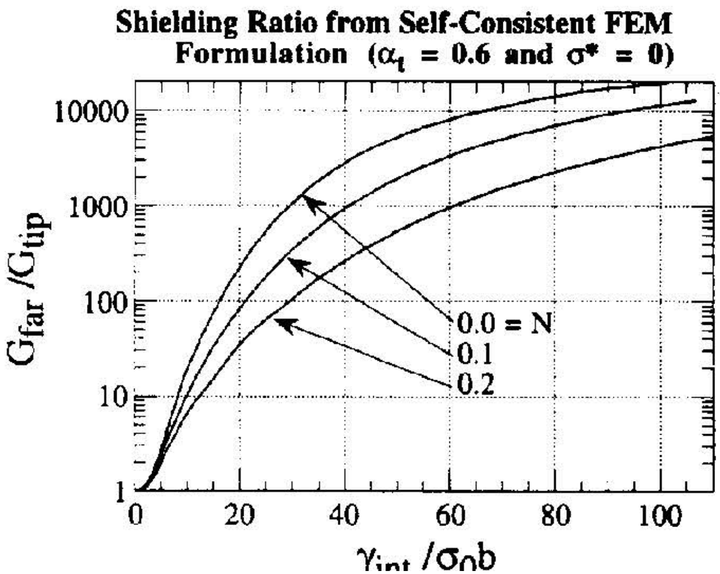
[1]Department of Mechanical and Environmental Engineering, University of California, Santa Barbara, CA 93106-5070, [2]Division of Engineering and Applied Sciences and Department of Earth and Planetary Sciences, Harvard University, Cambridge, MA 02138 and [3]Division of Engineering, Brown University, Providence, RI 02912, U.S.A.

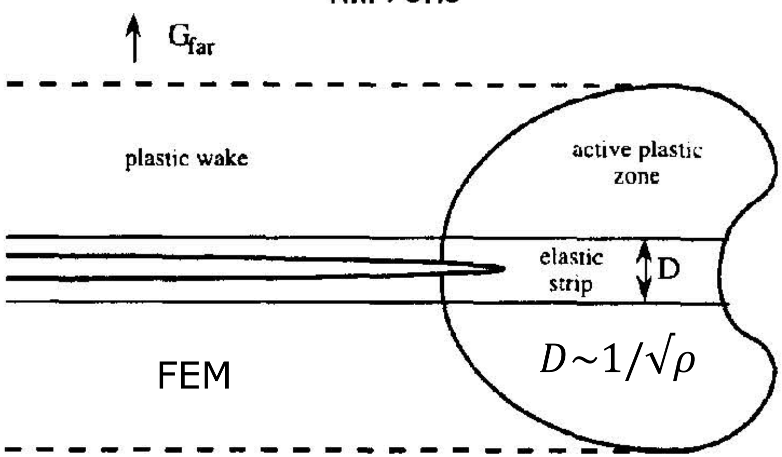
Abstract—A theory is proposed for cleavage in the presence of plastic flow, in circumstances which do not involve strong viscoplastic retardation of dislocation motion. We build upon recent notions that recognize the large disparity between relevant length scales involved in plastic flow processes around cracks in metals and on metal–ceramic interfaces.

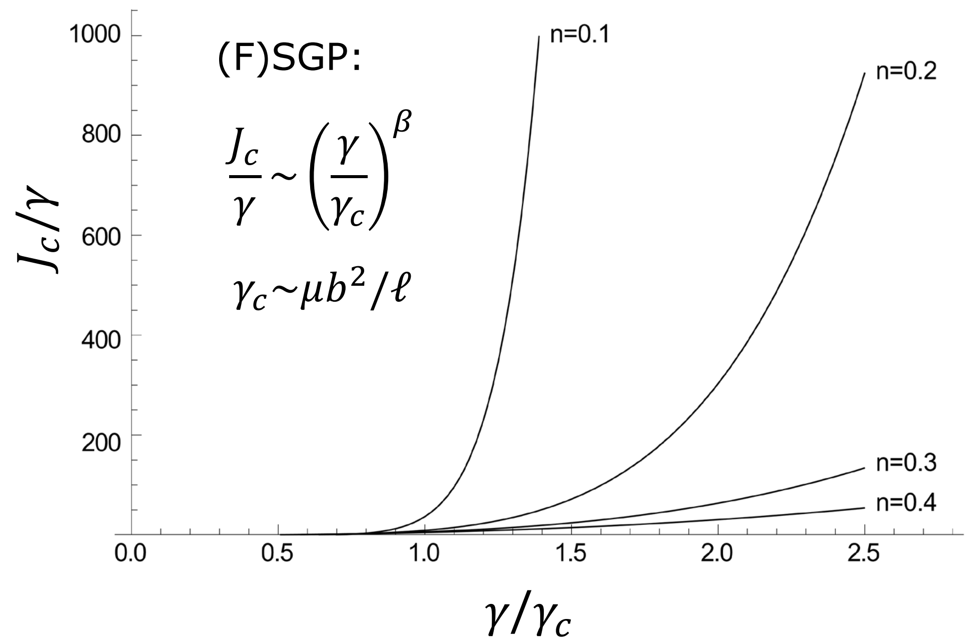
...

For steady-state crack growth to occur, it is found that the applied energy release rate G must generally be several orders of magnitude larger than the ideal work necessary to separate the interface, at least when D is taken as dislocation spacing. Furthermore, this "shielding" ratio is found to be strongly sensitive to the ideal work of fracture itself, as well as other material properties. *Copyright © 1996 Acta Metallurgica Inc.*

What have we learned?







- J_c rises sharply above γ, provided $\gamma > \gamma_c$ (threshold)
- γ has gating effect on J_c
- (F)SGP + work hardening exponents < 1, explain ductile fracture, scaling

Beltz, G., Rice, J.R., Shih, C.F. & Xia, L., *Acta Mater*., **44**(10) (1996) 3943-3954.

Ariza, M.P, Conti, S. & Ortiz, M., Eur J Mech A Solids (submitted).

Concluding remarks

Thank you!